FOURIER ANALYSIS ON CLASSICAL GROUPS

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We look at the following groups: the additive group \( \mathbb{R} \) of the real numbers; the additive group \( \mathbb{Z} \) of the integers [1, p. 12–13]; the additive group of the complex numbers of absolute value 1, and the additive group \( \mathbb{T} \) of the real numbers modulo 2\( \pi \), or equivalently, the circle group \( \mathbb{T} \), the multiplicative group of the complex numbers of absolute value 1, and the additive group \( \mathbb{Z}/2\mathbb{Z} \) of the integers [1, p. 12–13].

The Fourier series of the probability measure \( P \) on \( \sigma \)-algebra \( \mathcal{B}(\mathbb{T}) \) is defined by \( \hat{P}(p) = \int_{\mathbb{T}} e^{ipz} dP(z) \), where \( e^{ipz} = e^{ip} \), \( p \) is an integer [2, p. 50]. The circle \( \mathbb{T} \) can be rolled out onto the interval \([0, 1]\) by means of the correspondence \( e^{2\pi i x} \leftrightarrow x \in [0, 1) \).

Suppose \( \xi \) is a random variable taking values in \( \mathbb{R} = (-\infty; \infty) \). We prove the Lebesgue decomposition on the group \( \mathbb{S} \).

**THEOREM 1.** Let \( c_1 + c_2 + c_3 = 1 \), \( c_i \geq 0 \), \( i = 1, 2, 3 \). For each \( p = 0, \pm 1, \pm 2, \ldots \), it is true

\[
P_0(p) = c_1 \hat{P}_{\Theta_0}(p) + c_2 \hat{P}_{\Theta_1}(p) + c_3 \hat{P}_{\Theta_3}(p),
\]

where \( \Theta = (2\pi/|\beta|) \xi \ (\text{mod } 2\pi) \), \( l \geq 1 \), \( \beta > 0 \), and \( \Theta_i \), \( i = 1, 2, 3 \), are random angles and

\[
\hat{P}_{\Theta_0}(p) = \int_0^{2\pi} e^{ip\theta} \frac{|\beta|}{2\pi} \mathbb{P}\left( \frac{3\beta}{2\pi} \left( \theta + 2k\pi \right) \right) d\theta,
\]

\[
\hat{P}_{\Theta_1}(p) = \sum_{r=0}^{l-1} e^{ipr} \frac{|\beta|}{2\pi} \mathbb{P}\left\{ \xi_2 = \frac{|\beta|}{2\pi} \left( r + lp \right) \right\},
\]

\[
\hat{P}_{\Theta_3}(p) = \int_0^{2\pi} e^{ip\theta} \sum_{k=-\infty}^{\infty} \mathbb{P}\left\{ \beta k + \xi_3 < \frac{|\beta|}{2\pi} \left( \theta + 2k\pi \right) \right\} d\theta.
\]

**References**