ELEMENT-WISE SHRINKAGE OF POSITIVE CORRELATION MATRICES FOR INCREASING SPARSITY

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This research is a contribution to the study of correlation matrices particularly with respect to sparse matrices. We are interested in methods which shrink empirical correlations towards zero to produce a sparse matrix, which better reveals structure. The available methods fall broadly into those which act on the whole matrix, for example, via analysis of eigenvalues, and methods which act entrywise, that is, more directly on each correlation. Our method here falls into the second category and the computational cost is proportional to the number of entries in the matrix.

We shall use the notation \( C = [c_{ij}] \) for a correlation matrix defined as an \( n \times n \) positive definite matrix with ones on the diagonal. We shall assume throughout that all \( c_{ij} \geq 0 \). By shrinkage we shall mean that we possess a univariate function \( g(\cdot) \) and replace every entry \( c_{ij} \) by \( \tilde{c}_{ij} = g(c_{ij}) \). We shall use the notation \( g(C) \) for the matrix \( [g(c_{ij})] \). The nature of the function \( g(\cdot) \) and its effects are the main topic of this research. We start with some basic requirements: (i) \( g(y) \) is non-decreasing for \( y \in [0, 1] \), (ii) \( g(y) \leq y, y \in [0, 1] \), (iii) \( g(0) = 0, g(1) = 1 \). If (i)–(iii) are satisfied, we can say (iv) there is a \( \delta, 0 \leq \delta \leq 1 \), such that \( g(y) = 0 \) only if \( y \in [0, \delta] \).

We summarise some basic properties we shall require of our method.

P1. The method operates entrywise. This is covered by the same function \( g(y) \) being applied to every entry: two correlations with the same value are treated in the same way.

P2. The method increases, or does not decrease, sparsity. Measuring sparsity by the number of zeros (condition (iv) above) guarantees this.

P3. \( g(C) \) is nonnegative definite.

P4. \( g(C) \) is closer to the identity matrix than \( C \) in some sense.

We will explain how to measure the closeness of \( C \) to the identity matrix and how to obtain such a function \( g \) in the talk.

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