DUALLY FLAT STRUCTURES FOR $q$-GAUSSIAN FAMILIES

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A $q$-Gaussian distribution is a generalization of an ordinary Gaussian distribution. It was introduced in anomalous statistical physics. The set of $q$-Gaussian distributions include many useful probability densities such as an ordinary Gaussian distribution, a Cauchy distribution, a Student’s $t$-distribution and a semi-circle distribution.

In information geometry, it is known that the set of ordinary Gaussian distributions is regarded as a Riemannian manifold with dually flat affine connections. More generally, an exponential family is regarded as a dually flat space [1]. In the $q$-Gaussian case, though a $q$-Gaussian family can be regarded as a statistical manifold or a dually flat space, its structure is not unique. However, this variety of geometric structures characterizes the geometry of $\alpha$-divergences, $\beta$-divergences, and normalized Tsallis relative entropies [3].

Recently, Mori [5] showed that a $q$-Gaussian family admits dually flat affine connections in the sense of the ordinary Gaussian family. This fact implies that the $q$-Gaussian family naturally has another dually flat structure which has the invariant property in information geometry.

In this presentation, we give the dually flat structure of the $q$-Gaussian family under Mori’s dual affine connections. To elucidate the relations of another dually flat structures, we consider the sequential structure of escort expectations [2, 4].

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References