WAVELET MONTE CARLO – A NEW SAMPLING METHOD

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We present Wavelet Monte Carlo (WMC) – a novel algorithm that is able to produce independent samples from a non-standard distribution of interest when a direct sampling approach is not viable. The WMC combines the theory of wavelets and survival analysis to produce an original sampling method that could be an alternative to more popular MCMC methods.

The algorithm is constructed such that we start with a sample from a starting distribution \( x_0 \sim f(x) \) and gradually transform the sample to one from a target distribution \( y \sim g(x) \). The transformation involves new points \( x_t \) being sampled via mother wavelets \( \psi_{ji}(x) \) from the intermediate distributions \( f_t(x) = f(x) + t[g(x) - f(x)] \), where \( t \in [0, 1] \) is an artificial time parameter that indexes all intermediate distributions between \( f(x) \) and \( g(x) \) in the linear form. Furthermore, a survival time is being sampled for each new sample point \( x_t \) to transition the time parameter \( t \) until \( t = 1 \). If the sampled survival time for a point \( x_t \) is \( t \geq 1 \), we have reached our target distribution \( g(x) \) and have produced a sample point from our target, \( x_t \sim g(x) \).

Theoretical analysis of the WMC algorithm guarantees not only that samples produced by the algorithm are independent of each other but also that none of them are lost during the process, starting with a sample of size \( N \) from the distribution with density \( f(x) \) from a starting distribution, we are guaranteed to produce \( N \) realisations from our target. Furthermore, as long as the functional form of the target is known, there is no issue of having a multi-modal target distribution, so the algorithm never gets stuck in local modes, and does not face an issue of exploring a space well enough. Due to an absence of any sort of chain involved in the steps that are needed to produce separate samples from the target, quite trivial parallelisation techniques could be used to utilise multiple cores of the CPU or GPU used during the computation.