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IMPROVED BOUNDS FOR SQUARE-ROOT LASSO AND SQUARE-ROOT SLOPE

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Extending the results of Bellec, Lecué and Tsybakov [1] to the setting of sparse high-dimensional linear regression with unknown variance, we show that two estimators, the Square-Root Lasso and the Square-Root Slope, can achieve the optimal minimax prediction rate, which is $(s/n)\log(p/s)$, up to some constant, under some mild conditions on the design matrix. Here, n is the sample size, p is the dimension and s is the sparsity parameter. We also prove optimality for the estimation error in the l_q-norm, with $q \in [1,2]$ for the Square-Root Lasso, and in the l_2 and sorted l_1 norms for the Square-Root Slope. Both estimators are adaptive to the unknown variance of the noise. The Square-Root Slope is also adaptive to the sparsity s of the true parameter. Next, we prove that any estimator depending on s, which attains the minimax rate, admits an adaptive to s version still attaining the same rate. We apply this result to the Square-Root Lasso. Moreover, for both estimators, we obtain valid rates for a wide range of confidence levels, and improved concentration properties as in [1] where the case of known variance is treated. Our results are non-asymptotic.

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