Consider a study variable \( x \) taking real values in \( X = \{ x_1, \ldots, x_N \} \) in the population \( U = \{ 1, \ldots, N \} \); \( \mathbb{X} = \{ X_1, \ldots, X_n \} \) is the set of measurements of the simple random sample units \( \{ 1, \ldots, n \} \), \( n < N \), drawn without replacement from \( U \). The \( L \)-statistic
\begin{equation}
L = L_n(\mathbb{X}) = \frac{1}{n} \sum_{j=1}^{n} c_{j,n} X_{j,n}
\end{equation}
is a linear combination of the order statistics \( X_{1,n} \leq \cdots \leq X_{n,n} \) of \( \mathbb{X} \) with the real coefficients \( c_{j,n} = J(j/(n+1)) \), where \( J : (0,1) \to \mathbb{R} \), called weights. The sample mean, Gini’s mean difference statistic and trimmed means are particular cases of \( L \).

We aim to estimate the distribution function
\begin{equation}
F_S(y) = \mathbb{P}\{ \hat{\sigma}_L^{-1}(L - E(L)) \leq y \}
\end{equation}
of the Studentized \( L \)-statistic, where
\begin{equation}
\hat{\sigma}_L^2 = \hat{\sigma}_L^2(\mathbb{X}) = \left( 1 - \frac{n}{N} \right) \frac{n-1}{n} \sum_{k=1}^{n} (L(k) - L)^2, \quad L = \frac{1}{n} \sum_{k=1}^{n} L(k),
\end{equation}
is the jackknife estimator of the variance \( \sigma_L^2 = \text{Var}(L) \). Here \( L(k) = L_{n-1}(\mathbb{X}\setminus\{X_k\}) \), \( 1 \leq k \leq n \), are \( L \)-statistics with the weights \( c_{j,n-1} = J(j/n) \), \( 1 \leq j \leq n-1 \).

Usually, empirical Edgeworth expansions (EEE) and bootstrap approximations improve the normal approximation applied to \( F_S(y) \), if the sample size is not large enough. If well-correlated auxiliary information is available for all units of \( U \), the calibration technique \cite{DevilleSarndal1992} corrects EEEs based on the sample \( \mathbb{X} \) only \cite{PumputisCiginas2013}. We construct a new calibrated nonparametric bootstrap approximation to \( F_S(y) \), and compare it with selected calibrated (parametric) EEEs and some other approximations in a simulation study.

References