REGULAR VARIATION AND HEAVY-TAIL LARGE DEVIATIONS FOR TIME SERIES

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The goal of this lecture is to present some of the recent results on heavy-tail modeling for time series and the analysis of their extremes.

Over the last 10–15 years research in extreme value theory has focused on the interplay between the serial extremal dependence structure and the tails of time series. In this context, heavy-tailed time series (as appearing in finance, climate research, hydrology, and telecommunications) have been studied in detail, leading to an elegant probabilistic theory and statistical applications.

Heavy tails of the finite-dimensional distributions are well described by multivariate regular variation: it combines power-law tails of the marginal distributions and a flexible dependence structure which describes the directions at which extremes are most likely to occur; see Resnick (2007) [6] for an introductory text to multivariate regular variation.

A second line of research has continued through the years but attracted less attention: heavy-tail large deviations. In the 1960s and 1970s A.V. and S.V. Nagaev [4, 5] started studying the probability of the rare event that a random walk with i.i.d. heavy-tailed step sizes would exceed a very high threshold far beyond the normalization prescribed by the central limit theorem. In the case of subexponential (in particular regularly varying) distributions, the tail of the random walk above high thresholds is essentially determined by the maximum step size. Later, related results were derived for time series models by Davis and Hsing (1995) [1], Mikosch and Wintenberger (2014, 2016) [2, 3], among others. Here the main difficulty is to take into account clustering effects of the random walk above high thresholds.

Regular variation and heavy-tailed large deviations are two aspects of dependence modeling in an extreme world. They are similar in the sense that they are closely related to the weak convergence of suitable point processes. Actually, both regular variation and heavy-tail large deviations are defined via the vague convergence of suitably scaled probability measures whose (infinite) limit measure has interpretation as the intensity measure of a Poisson process. In the heavy-tailed time series world this relationship opens the door to the Poisson approximation of extreme objects such as the upper order statistics of a univariate sample, the largest eigenvalues of the sample covariance matrix of a very high-dimensional time series, and to functionals acting on them.

References


