GROWTH-FRAGMENTATION PROCESSES

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Pure fragmentation processes have been introduced by A.N. Kolmogorov in 1941, as models for an inert mass which undergoes repeatedly random dislocations. To deal with dynamics which are mathematically tractable, one assumes the branching property, that is, that different fragments evolve independently. Further, one focuses on a self-similar setting, in the sense that the statistics of the process starting at time 0 from a single mass \( m > 0 \) can be reduced up to a simple scaling transformation to that for a unit mass \( m = 1 \). The distribution of a pure fragmentation process is then determined by the index of self-similarity and a so-called dislocation measure that encodes the statistics of the sudden dislocations.

Growth-fragmentation processes can be thought of as fragmentation processes to which a growth phenomenon has been incorporated. They have been introduced in Life Science as models of populations of cells or bacteria, which evolve by growth and division. They have also appeared more recently in the framework of random planar geometry.

The incorporation of growth changes fundamentally the dynamics of fragmentation processes. For instance, growth may “compensate” dislocations and avoid instantaneous shattering despite of an extremely high intensity of dislocations. On the other hand, growth may also induce “local” explosions, in the sense that an infinite number of macroscopic particles may be present at the same time.

This lecture is a survey of some recent advances in this topic: I will describe the general construction of (self-similar) growth-fragmentation processes in terms of a cell system, whose evolution is related to that of a Lévy process without positive jumps. In particular, negative jumps of the latter are interpreted as birth events for the cell system. I will also discuss the connection with branching random walks, and some remarkable martingales that arise in this framework. In particular, this leads to an important area measure, which plays a central role in a variety of limit theorems.