POLYNOMIAL EQUIVALENCE OF THE KULLBACK DIVERGENCE FOR EXPONENTIAL FAMILIES

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Watanabe et al. proved in 2004 that the Kullback information of Gaussian mixtures is not analytic at points on the boundary of the parameter space where one of the mixing parameters is zero [4]. This nonanalyticity is problematic for statistical theorems which make this seemingly mild assumption [3, §7.8]. To overcome the problem, the authors showed using a resolution of singularities that the Kullback information is equivalent to the squared distance between the true distribution and the model distribution, and that this equivalence is usually sufficient for statistical theorems. Here, two nonnegative functions $f, g : \Omega \rightarrow \mathbb{R}_{\geq 0}$ are equivalent if there exist positive constants $c_1, c_2 > 0$ such that $c_1 f(\omega) \leq g(\omega) \leq c_2 f(\omega)$ for all $\omega \in \Omega$. In this talk, we generalize their results to mixtures of exponential families, and show that for polynomial families [1] the Kullback information is in fact equivalent to a polynomial, namely, the sum of squares of differences between the true moments and the model moments up to some order. Consequently, the asymptotic behavior of learning algorithms may be unraveled using ideal-theoretic strategies from algebraic geometry [2]. This work was initiated through a discussion between the authors at the Oberwolfach Workshop on Algebraic Statistics in April 2017.

References


