DYNAMIC EMPIRICAL QUANTILES FOR TIME SERIES

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The empirical quantiles for independent data provide a good summary of the distribution of the observations and are very useful for plotting them. However, the empirical quantiles for time series have some drawbacks to be used as summary of a set of time series as they only show the marginal distribution at every time point but do not give any information about the time series dynamics. Thus, sets of stationary time series will have empirical pointwise quantiles that will be approximately parallel lines for all time points \( t \). In this article we propose a new definition of empirical time series quantiles that tries to address this limitation. We will call dynamic empirical quantiles those we define, to emphasize their empirical nature. We prove that, in the limit, the empirical dynamic quantiles converge to the pointwise quantiles, but in real time series they will provide very different information about the time series dynamics. A fast procedure to compute the dynamic quantiles is presented, as well as several plots for high-dimensional time series using these dynamic quantiles. The procedure is illustrated by several examples.