CLUSTERING AND DEGREE IN AFFILIATION NETWORKS

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Mathematical modeling of complex networks aims at explaining and reproduction of characteristic properties of large real world networks. Here we focus on the clustering property meaning by this the tendency of nodes to cluster together by forming relatively small groups with a high density of ties within a group. In particular, we are interested in the correlation between clustering and degree explained below. Locally, in a vicinity of a vertex, clustering can be measured by the local clustering coefficient, the probability that two randomly selected neighbors of the vertex are adjacent. The average local clustering coefficient across vertices of degree \( k \), denoted \( C(k) \), for \( k = 2, 3, \ldots \), describes the correlation between clustering and degree. Empirical studies of real social networks show that the function \( k \rightarrow C(k) \) is decreasing. Moreover, in the film actor network \( C(k) \) obeys the scaling \( k^{-1} \), in the internet graph it obeys the scaling \( k^{-0.75} \). We are interested in modeling and explaining the scaling \( k^{-\delta} \), for \( \delta > 0 \).

Clustering in an affiliation network can be explained by the auxiliary bipartite structure defining the adjacency relations between actors: every actor is prescribed a collection of attributes and any two actors sharing an attribute have positive chances of being adjacent. The respective random intersection graph \( G \) on the vertex set \( V = \{ v_1, \ldots, v_n \} \) and with the auxiliary set of attributes \( W = \{ w_1, \ldots, w_m \} \) defines adjacency relations between vertices with the help of a random bipartite graph \( H \) linking actors to attributes: the pairs of vertices sharing a common neighbor in \( H \) have high chances to be adjacent in \( G \). Random intersection graphs admit tunable power law degree distribution, non-vanishing global clustering coefficient, short typical distances. The talk addresses yet another remarkable feature of random intersection graphs, the tunable scaling \( k^{-\delta} \), \( \delta \geq 0 \), of respective conditional probability \( C_G(k) = P(v_2 \sim v_3 | v_2 \sim v_1, v_3 \sim v_1, d(v_1) = k) \), the theoretical counterpart of \( C(k) \). The argument uses a first order asymptotics of the local probabilities of randomly stopped sums \( P(X_1 + \cdots + X_N = t) \sim at^{-\alpha} \) as \( t \to +\infty \) in the case where the i.i.d. summands \( X_1, X_2, \ldots \) and the number \( N \) of summands are independent power law lattice random variables [1].

References