SENSITIVITY OF ROUGH DIFFERENTIAL EQUATIONS

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Introduced by T. Lyons [2], the theory of rough paths allows one to define differential equations driven by irregular paths, that is, solutions of equations of the type
\[ y_t = a + \int_0^t f(y_s) \, dx_s \]
for a continuous but irregular path \( x \) with values in a Banach space, a point \( a \) and a family \( f \) of regular vector fields. Such integrals allow one to solve stochastic differential equations when the path \( x \) is a trajectory of a random process, at the price of extending it as a path \( x \) living in a bigger, non-commutative space. The path \( x \) is called a rough path.

From its very construction, the Itô map \( J : x \mapsto y \) is Lipschitz continuous. This feature is of primary importance for studying regularizations, numerical approximations, as well as diverse applications to modelling. Several works consider higher regularity of the Itô map with respect to \( a \) (starting point), \( x \) (driving path) and \( f \) (vector field). We present here an approach [1] which unifies the previous approaches under weaker conditions. For this, we consider \( y \) as a solution to a fixed point problem \( y = \mathcal{I}(\mathcal{O}(f,y),a,x) \) where \( \mathcal{O}(f,y) = f(y) \) and \( \mathcal{I}(z,x) \) is the linear integral of \( z \) against \( x \). Combining the regularity of \( y \mapsto \mathcal{O}(f,y) \) for paths of finite \( p \)-variation with the Implicit Function Theorem in Banach spaces, the Itô map \( J \) is shown to be locally \( \mu \)-Hölder continuous with respect to all its parameters \( (a,f,x) \), where \( \mu > 1 \) depends on the regularity of the vector field and the driving paths. With respect to similar results established for ordinary differential equations, the topology induced by the choice of the \( p \)-variation, a crucial choice for establishing the continuity of the Itô map, implies a loss of regularity.

References
