A PARTICLE MODEL FOR WASSERSTEIN TYPE DIFFUSION

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The discussion will be devoted to a family of interacting particles on the real line which have a connection with the geometry of Wasserstein space of probability measures. We will consider a physical improvement of a classical Arratia flow, but now particles can split up and they transfer a mass that influences their motion [1–4]. The particle system can be also interpreted as an infinite-dimensional version of sticky reflecting dynamics on a simplicial complex. If \( X_t(u) \) denotes the position of particle \( u \in (0, 1) \) at time \( t \geq 0 \), then the model appears as a solution to the infinite-dimensional SDE with discontinuous coefficients [1, 3, 4]:

\[
\begin{align*}
\frac{dX_t}{dt} &= \text{pr}_f dW_t + (\xi - \text{pr}_f \xi) dt
\end{align*}
\]

in the subspace of non-decreasing functions from \( L_2 := L_2([0, 1], du) \), where \( \text{pr}_f \) is the projection in \( L_2 \) on the linear subspace of \( \sigma(f) \)-measurable functions and \( dW \) is an \( L_2 \)-white noise. The non-decreasing function \( \xi \) is responsible for the reflection of particles.

The existence of a weak solution to the SDE can be constructed using, e.g., a finite particle approximation [1, 2]. In the talk, we are going to consider a reversible case, where the construction is based on a new family of measures on the set of real non-decreasing functions as reference measures for naturally associated Dirichlet forms. In this case, the intrinsic metric leads to a Varadhan formula [3, 4]

\[
P\{\mu_t = v\} \sim e^{-\frac{d_{W}^2(\mu_0, v)}{2t}}, \quad t \ll 1,
\]

for the short time asymptotics with the Wasserstein metric \( d_W \) for the associated measure valued diffusion \( \mu_t = \text{Leb} \circ X_t^{-1} \). The talk is based on joint work with Max von Renesse.

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References