LOCAL TIMES ON SURFACES FOR BROWNIAN MOTIONS ON CARNOT GROUPS

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We study a class of processes, known as Brownian motions on Carnot groups, which are important for general theory of diffusions (see [1] for more details). In particular, we are interested in their behaviour with regard to arbitrary surface, or more precisely a local time w.r.t. surface measure of this surface. If \( \{X_t, t \in [0,1]\} \) is a measurable stochastic process, then we define approximations for local time w.r.t. measure \( \nu \) as 

\[
\gamma_\varepsilon = \frac{1}{0} f_\varepsilon(X_t) \, dt \quad \text{for } \varepsilon > 0,
\]

where \( f_\varepsilon \) is a family of non-negative functions such that the measure \( f_\varepsilon(x) \, dx \) converges weakly as \( \varepsilon \to 0^+ \) to the measure \( \nu \). For elliptic diffusions it is known that for surfaces of codimension 1 the local time does exist as a limit of \( \gamma_\varepsilon \) in \( L^2 \) (under rather general conditions). To study local times w.r.t. surface measures for Brownian motions on Carnot groups, we use the bounds for their densities, which can found in [2] and [3]. It turns out that for Brownian motions on Carnot groups even linear subspaces of codimension 1 may not admit local time in the sense of convergence of \( \gamma_\varepsilon \) in \( L^2 \). We found several cases when local time exists and few cases when it does not. This question is related to the geometry of the balls w.r.t. the natural distance in Carnot groups, and to answer it, uniform bounds on the surface measure of such balls were investigated, improving on some known results and ideas from [4].

References