HYPOTHESIS TESTING AND PARAMETER ESTIMATION FOR FRACTIONAL ORNSTEIN–UHLENBECK PROCESS

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We study the inference problem for the fractional Ornstein–Uhlenbeck process $X = \{X_t, t \geq 0\}$, which is the unique solution of the stochastic differential equation

$$dX_t = \theta X_t \, dt + dB^H_t, \quad X_0 = x_0 \in \mathbb{R}. \quad (1)$$

Here $\theta \in \mathbb{R}$ is an unknown drift parameter. The noise is modelled as a fractional Brownian motion $B^H = \{B^H_t, t \geq 0\}$ with a known Hurst index $H \in (0, 1)$. First, we investigate the problem of the drift parameter estimation by continuous and discrete observations of the trajectory of $X$. We construct several estimators and prove their strong consistency. It turns out that the methods for constructing the estimators and their asymptotic properties substantially depend on the sign of the unknown parameter. This motivates testing the hypothesis about the sign of the drift parameter in the model (1). We propose a comparatively simple test for testing the null hypothesis $H_0 : \theta \leq 0$ against the alternative $H_1 : \theta > 0$ and prove its consistency. Contrary to the previous works, our approach is applicable for all $H \in (0, 1)$. The test is based on the observations of the process $X$ at two points, 0 and $T$. The distribution of the test statistic is computed explicitly, and the power of the test can be found numerically for any given simple alternative. Also we consider testing the hypothesis $H_0 : \theta \geq \theta_0$ against $H_1 : \theta \leq 0$, where $\theta_0 \in (0, 1)$ is some fixed number. As an auxiliary result of independent interest we compute the covariance function of the fractional Ornstein–Uhlenbeck process.

References