SPATIAL PROPERTIES OF ONE-DIMENSIONAL BROWNIAN FLOWS

E.V. GLINYANAYA

Institute of Mathematics, National Academy of Sciences of Ukraine, Kiev

e-mail: glinkate@gmail.com

DEFINITION ([11]). The Harris flow with the local characteristic $\Gamma$ is a family $\{x(u, \cdot), u \in \mathbb{R}\}$ of Brownian martingales with respect to the joint filtration such that:

1) for every $u \in \mathbb{R}$, $x(u, 0) = u$;
2) for every $u_1, u_2 \in \mathbb{R}$, $u_1 \leq u_2$, $t \geq 0$, $x(u_1, t) \leq x(u_2, t)$;
3) for every $u_1, u_2 \in \mathbb{R}$, $d \langle x(u_1, \cdot), x(u_2, \cdot) \rangle(t) = \Gamma(x(u_1, t) - x(u_2, t)) dt$.

REMARK. In [1] the existence of $x$ is proved for real continuous positive definite function such that $\Gamma(0) = 1$ and $\Gamma$ is Lipshits outside any neighborhood of zero. In the case when $\Gamma = 1_{[0]}$ the existence of $x$ was proved by R. Arratia [2], and the flow was called by his name.

Depending on the properties of $\Gamma$, the coalescence of particles can happen. We are interested in the asymptotic properties of obtained clusters and their number. The key tool in our investigation is a mixing property of the flow with respect to the spatial variable.

THEOREM 1 ([3]). The process $\{x(u, t) - u, u \in \mathbb{R}\}$ is stationary and, under the condition $\Gamma(u) \to 0$ as $|u| \to \infty$, has the mixing property.

THEOREM 2 ([3]). Let $\text{supp} \Gamma \subset [-c, c], c > 0$. Then for the strong mixing coefficient $\alpha$ of the process $\{x(u, t) - u, u \in \mathbb{R}\}$ we have $\alpha(h) \leq 2 \sqrt{\frac{2}{\pi}} \int_{h-c}^{h+c} e^{-x^2/2} dx$.

Using the central limit theorem for a stationary sequence proved in [4], one can get the asymptotic distribution for the normalized number of clusters in the Arratia flow.

THEOREM 3. Let $\Gamma = 1_{[0]}$ and $\nu([u_1, u_2]) = \#x([u_1, u_2], t)$.

Then, for every $t > 0$, $\sqrt{n} \nu([0, u]) \to N(0, \sigma^2_t)$, as $n \to \infty$, where $\sigma^2_t = \frac{2t}{\sqrt{n}}$.

COROLLARY 1. Let $\Gamma = 1_{[0]}$. Then $\sqrt{t} \nu([0, 1]) - \frac{1}{\sqrt{t} \sqrt{n}} \to N(0, \sigma^2_t)$ as $t \to 0$.

References

1. T.E. Harris, Coalescing and noncoalescing stochastic flows in $\mathbb{R}_1$, Stochastic Processes and their Applications 17, 1984, pp. 187–210. MR0751202