WAVE EQUATION WITH GENERAL STOCHASTIC MEASURE COLORED IN TIME

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Let $B$ be the Borel $\sigma$-algebra of $[0,T]$, $T > 0$, $L_0(\Omega, \mathcal{F}, P)$ be the set of real-valued random variables defined on a complete probability space $(\Omega, \mathcal{F}, P)$.

Consider the Cauchy problem of the stochastic wave equation

$$\begin{cases}
\frac{\partial^2 u(t, \vec{x})}{\partial t^2} = a^2 \Delta \vec{x} u(t, \vec{x}) + f(t, \vec{x}, u(t, \vec{x})) + \sigma(t, \vec{x}) \dot{\mu}(t), \\
u(0, \vec{x}) = u_0(\vec{x}); \quad \frac{\partial u(0, \vec{x})}{\partial t} = v_0(\vec{x}),
\end{cases}$$

where $(t, \vec{x}) \in [0,T] \times \mathbb{R}^d$, $d = 1,2$, $a > 0$, $\Delta \vec{x}$ is Laplace operator, $u(t, \vec{x}) = u(t, \vec{x}, \omega) : [0,T] \times \mathbb{R}^d \times \Omega \rightarrow \mathbb{R}$ is an unknown measurable random function. Here $\mu$ is a general stochastic measure defined on $B$, i.e. $\mu : B \rightarrow L_0(\Omega, \mathcal{F}, P)$ is a $\sigma$-additive mapping. We do not assume existence of moments, positivity or martingale property for $\mu$.

We investigate the mild solution of (1), i.e., any measurable random function $u(t, \vec{x}) = u(t, \vec{x}, \omega) : [0,T] \times \mathbb{R}^2 \times \Omega \rightarrow \mathbb{R}$ such that $\forall (t, \vec{x})$

$$u(t, \vec{x}) = \int_{\mathbb{R}^d} S_d(t, \vec{x} - \vec{y}) u_0(\vec{y}) d\vec{y} + \frac{\partial}{\partial t} \left( \int_{\mathbb{R}^d} S_d(t, \vec{x} - \vec{y}) u_0(\vec{y}) d\vec{y} \right) + \int_0^t ds \int_{\mathbb{R}^d} S_d(t-s, \vec{x} - \vec{y}) f(s, \vec{y}, u(s, \vec{y})) d\vec{y} + \int_{[0,t]} d\mu(s) \int_{\mathbb{R}^d} S_d(t-s, \vec{x} - \vec{y}) \sigma(s, \vec{y}) d\vec{y}.$$

Here $S_d(t, \vec{x})$ is the fundamental solution of the wave equation.

We assume some conditions on functions $f, \sigma, u_0, v_0$ like measurability, boundedness and regularity.

The existence, uniqueness and Hölder continuity of the mild solution are proved.

The case $d = 1$ is presented in [1].

References