CODING OF POISSON RANDOM SETS: LARGE DEVIATIONS

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Consider a random set (or "picture") $S := \bigcup \mathcal{B}(\xi_i, R_i)$ where $\{\xi_i\}$ is a Poisson point configuration in the $d$-dimensional cube $[0, 1]^d$, $\{R_i\}$ are non-negative i.i.d. random variables independent of the configuration and $\mathcal{B}(\xi, R)$ denotes the ball of radius $R$ centered at $\xi$ with respect to some norm in $\mathbb{R}^d$. Let $K$ be the minimal number of balls needed in order to reproduce $S$.

We study large deviation probabilities for $K$ and prove in some cases that

$$\Pr(K \geq n) = \exp\{-an\ln n(1 + o(1))\}, \quad \text{as } n \to \infty,$$

where the constant $a$ may explicitly depend on $d$, on the distribution of radii, and on the norm under consideration. In many cases the problem of finding the value of $a$ remains open although some upper and lower bounds are available.

This asymptotics has natural corollaries in high dimensional quantization problems, cf. [1] and [2].

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References
