LÉVY PROCESSES WITH RESPECT TO THE INDEX WHITTAKER CONVOLUTION

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It was recently discovered that the product of two Whittaker functions can be written in terms of an integral depending linearly on the Whittaker function itself. This product formula is related with the index Whittaker transform; it gives rise to a nonstandard convolution operation which induces a Banach algebra structure in the space of finite complex Borel measures on $[0, \infty)$ having the property that the convolution of probability measures is a probability measure.

In this talk we introduce and develop the notion of a Lévy process with respect to the index Whittaker convolution. We show that these processes constitute a family of Feller processes admitting a Lévy–Khintchine representation, and whose $L^2$-generator can be explicitly described. Martingale characterizations of this class of Lévy processes are presented.

In contrast with other generalized convolution structures studied in the literature, namely in the context of hypergroups, the Whittaker convolution is defined by a Cauchy problem which is not uniformly hyperbolic. Our results thus demonstrate that a nice theory of Lévy processes with respect to generalized convolutions can be developed even if the usual compactness assumption on the support of the convolution fails, shedding light into the connection between the properties of the convolution algebra and the nature of the singularities of the associated differential operator.

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