STOCHASTIC VERSIONS OF DINI’S THEOREM WITH APPLICATIONS TO STRONG APPROXIMATION OF SOLUTIONS TO SDES DRIVEN BY LÉVY PROCESSES

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Dini’s theorem describes one of the few situations in mathematics where pointwise convergence implies uniform convergence. The classical Itô–Nisio theorem can be viewed as a stochastic version of Dini’s theorem for series of independent symmetric processes with continuous paths over a metrizable compact set.

Many stochastic processes, however, live in nonseparable Banach spaces for which the Itô–Nisio theorem does not apply. This case includes Lévy processes under supremum or $\phi$-variation norms. We establish a generalization of Itô–Nisio theorem that covers such and other spaces in the spirit that pointwise convergence implies convergence in norm.

As applications we will consider pathwise approximations to solutions of stochastic differential equations driven by Lévy processes. We will discuss Itô map, which is just a solution to an ODE with a rough path input. Continuity of Itô map usually requires strong, nonseparable, Banach space norm on the path space. Our generalization of the Itô–Nisio theorem leads to strong pathwise convergence in certain Wiener subclasses of random polygonal lines (with jumps) to paths of a Lévy process. This, in turn, yields strong pathwise convergence of solutions of random differential equations to the solution of a stochastic differential equation driven by Lévy processes.

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