EXPECTED NUMBER OF VERTICES IN CONVEX HULLS OF RANDOM WALKS

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Consider an $n$-step random walk $(S_i)_{i=1}^n$ in $\mathbb{R}^d$ whose common distribution of increments assigns zero mass to hyperplanes. We obtain explicit distribution-free formulas for the expected number of $k$-dimensional faces (e.g., vertices for $k = 0$ and hyperfaces for $k = d - 1$) of the convex hull $C_n := \text{conv}(0, S_1, \ldots, S_n)$ of the random walk. The previous results of this type were obtained in early 1960s for $d = 2$ and $d = 3$ with $k = 2$.

The main ingredient of our computation is finding the probability that the origin of $\mathbb{R}^d$ is absorbed by the joint convex hull of several random walks and random walk bridges whose joint distribution of increments is invariant with respect to the action of a direct product of finitely many reflection groups of types $A$ and $B$. This probability, in turn, is related to the number of Weyl chambers of a product-type reflection group that are intersected by a linear subspace in general position. This approach was first used in [1], where we found the probabilities of absorption of the origin by convex hulls of a single random walk or a random walk bridge.

This talk is based on my joint work [2] with Zakhar Kabluchko (Münster) and Dmitry Zaporozhets (St. Petersburg).

References
