OPTION PRICING WITH FRACTIONAL STOCHASTIC VOLATILITY AND DISCONTINUOUS PAYOFF FUNCTION OF POLYNOMIAL GROWTH

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This is joint research with Viktor Bezborodov and Luca di Persio from the University of Verona. We consider the pricing problem related to payoffs of polynomial growth that can have discontinuities of the 1st kind. The asset price dynamic is modeled within the Black–Scholes framework characterized by a stochastic volatility term driven by a fractional Ornstein–Uhlenbeck process. In order to solve the aforementioned problem, we consider three approaches. The first one consists in a suitable transformation of the initial value of the asset price, in order to eliminate possible discontinuities. Then we discretize both the Wiener process and the fractional Brownian motion and estimate the rate of convergence of the related discretized price to its real value whose closed-form analytical expression is usually difficult to obtain. The second approach consists in considering the conditional expectation with respect to the entire trajectory of the fractional Brownian motion (fBm). Here we derive a representation for the option price which involves only an integral functional depending on the fBm trajectory, and then discretize the fBm and estimate the rate of convergence of the associated numerical scheme. In both cases the rate of convergence is the same and equals $n^{-rH}$, where $n$ is the partition size, $H$ is the Hurst index of the fBm, and $r$ is the H"older exponent of the volatility function. The third method consists in calculating the density of the integral functional depending on the trajectory of the fBm via Malliavin calculus and providing the option price in terms of the associated probability density.