CAPITAL INJECTIONS AND DIVIDENDS WITH TAX

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The surplus of an insurance portfolio is modelled by a spectrally negative Lévy process \( \{X_0^t\} \). We denote the initial capital by \( x = X_0^0 \). Dividends may be paid and, if the capital becomes negative, capital injections have to be made in order to keep the surplus positive. Using a dividend strategy \( \{D_t\} \) — an increasing process with \( D_0 = 0 \) — the surplus is \( X_D^t = X_0^t - D_t + L_t \), where the capital injections \( \{L_t\} \) is the smallest increasing process such that \( X_D^t \geq 0 \) for all \( t \).

In addition, tax has to be paid for dividends, but capital injections lead to exemptions from tax. Let \( 1 - \gamma \) be the tax rate, where \( \gamma \in (0, 1) \). We denote by \( Y_t \) the maximal amount of dividends that could be paid without tax. Letting \( Y_0 = y \), we have

\[
Y_t = y + L_t - \int_0^t \mathbb{1}_{Y_s > 0} \, dD_s^c - \sum_{s \leq t} \min\{\Delta D_s, Y_s - \}\,
\]

where \( D_t^c = D_t - \sum_{s \leq t} \Delta D_s \) is the continuous part of the dividend payments \( D_t \). The value of a dividend strategy \( \{D_t\} \) is

\[
V^D(x,y) = \mathbb{E}\left[ \int_0^\infty e^{-\delta t}(1_{Y_t > 0} + \gamma 1_{Y_t = 0}) \, dD_s^c + \sum_{t \geq 0} e^{-\delta t}\left[ \min\{\Delta D_t, Y_t - \}\right] + \gamma (\Delta D_t - Y_t - \right) + \eta \int_0^\infty e^{-\delta t} \, dL_t \right],
\]

where \( \delta > 0 \) is a preference parameter and \( \eta \geq 1 \). The value function is then \( V(x,y) = \sup_D V^D(x,y) \). Under the optimal strategy, an alternative interpretation of the tax is that the company has to pay tax whenever the surplus without dividends \( \{X_0^t\} \) is at a maximum, so that the income of the portfolio is diminished and less tax is paid.

We solve the problem and show that the optimal strategy is a two barrier strategy: if \( Y_t > 0 \), pay all capital above \( b_1 \) as dividend. If \( Y_t = 0 \), pay all capital above \( b_\gamma \) as dividend. Examples are the diffusion case treated in [2], the classical risk model treated in [1], or the perturbed risk model treated in [3].

References