TIME-INCONSISTENT STOPPING PROBLEMS

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Let $X$ be a diffusion with state space $E$. Let $f : E \to R$ and consider the classical problem of choosing a stopping time $\tau$ that maximizes

$$E_x(f(X_\tau)).$$

Recall that the solution to this problem is consistent in the sense that the stopping rule ‘stop when $X$ enters the stopping set’ is independent of the starting value $x$.

Now consider the following two problems:

1. Let $F : E \times E \to R$ and choose a stopping time $\tau$ that maximizes

$$E_x(F(X_\tau, x)).$$

2. Let $h : E \to R$ and $g : R \to R$, and choose a stopping time $\tau$ that maximizes

$$E_x(f(X_\tau)) + g(E_x(h(X_\tau))).$$

It is easy to see that both of these problems are in general time-inconsistent in the sense that the optimal rule for stopping will depend on the starting value $x$.

In the literature, there are two main ways of dealing with time-inconsistent control and stopping problems:

- The pre-commitment approach, which is to solve the problem for a fixed $x$ and allow the optimal solution to depend on $x$.
- The game-theoretic approach, which is to reinterpret the problem as a game and search for equilibrium solutions.

We have developed a framework for a game-theoretic approach to time-inconsistent stopping problems, including for example: (i) proper equilibrium definitions, (ii) mixed and pure stopping strategies, (iii) different necessary and sufficient equilibrium conditions, (iv) several examples, including mean–variance utility and endogenous habit formation problems.

Time-inconsistent problems were first studied in financial economics, in the context of endogenous habit formation, non-exponential discounting and mean–variance utility. These problems can be formulated within our framework.