COALESCING STOCHASTIC FLOWS ON $\mathbb{R}$

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Let $(\psi_{s,t})_{s\leq t}$ be a stochastic flow on $\mathbb{R}$, i.e. a family of measurable random mappings of $\mathbb{R}$ that act independently on non-overlapping intervals, satisfy stationarity condition $\psi_{s,t} = \psi_{0,t-s}$ (in distribution), and evolutionary property $\psi_{s,t} \circ \psi_{s,s+s'} = \psi_{s,s'+t}$. A stochastic flow $(\psi_{s,t})_{s\leq t}$ is called coalescing if there are $x \neq y$ such that the first meeting time $\tau_{x,y} = \inf\{t \geq 0 : \psi_{0,t}(x) = \psi_{0,t}(y)\}$ is finite with positive probability and for all $t \geq \tau_{x,y}$, $\psi_{0,t}(x) = \psi_{0,t}(y)$ [1].

We will study a possibility to represent a stochastic flow as a random dynamical system: $\psi_{s,t}(\omega, x) = \varphi(t - s, \theta_s \omega, x)$. Here $(\theta_h)_{h \in \mathbb{R}}$ is a measurable group of measure preserving transformations of the underlying probability space, $\varphi : \mathbb{R}_c \times \Omega \times \mathbb{R} \to \mathbb{R}$ is a measurable perfect cocycle over $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_h)_{h \in \mathbb{R}})$ [2]. Main result is formulated in terms of finite-point motions of the stochastic flow. Assume that $(\psi_{s,t})_{s\leq t}$ is a coalescing stochastic flow of mappings of $\mathbb{R}$, such that all its finite-point motions $t \to (\psi_{0,t}(x_1), \ldots, \psi_{0,t}(x_n))$ are continuous Feller processes. Further, assume that all random variables $\psi_{0,t}(x), t > 0, x \in \mathbb{R}$ are continuous and for any $t > 0, c < d$ there exists a continuous function $m_{c,d}$ such that for $c \leq x < y \leq d$ one has

$$\mathbb{P}(\forall s \in [0,t] \ c \leq \psi_{0,s}(x) < \psi_{0,s}(y) \leq d \leq m_{c,d}(y) - m_{c,d}(x)).$$

Theorem. [3] On a suitable probability space $(\Omega, \mathcal{F}, \mathbb{P}, (\theta_h)_{h \in \mathbb{R}})$ there exists a random dynamical system $\varphi$ such that $\psi_{s,t}(\omega, x) = \varphi(t - s, \theta_s \omega, x)$ is a version of the stochastic flow $(\psi_{s,t})_{s\leq t}$.

The result does not follow from well-known perfection results [2] because of spatial discontinuity of mappings $x \to \psi_{s,t}(x)$. Instead, we construct a suitable state space for coalescing stochastic flows that allows us to handle measurability questions. As applications, random dynamical systems for generalized Arratia flows and certain coalescing Harris flows are shown to exist, and ergodic theory for Arratia flows with drift is developed.

References