STABLE LIMITS FOR MARKOV CHAINS VIA THE PRINCIPLE OF CONDITIONING

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Our motivation comes from the paper by Jara, Komorowski and Olla [3], where a fractional diffusion was obtained as a scaled limit of functionals of Markov chains forming a probabilistic solution to a linear Boltzmann equation. The main tool used in [3] was a functional limit theorem on convergence to stable Lévy processes due to Durrett and Resnick [1] and the assumptions that made this functional limit theorem working were $L^2$-spectral gap and strong compactness properties of the Markov transition operator. In the particular example considered in [3] the ultraboundedness of the transition operator was used, but in the general considerations properties related to a weaker notion of hyperboundedness were assumed.

In the present paper we replace the hyperboundedness with a weaker notion of the uniform integrability in $L^2$ (2-UI in short) of the transition operator, a notion introduced in [4]. We provide an example of a Markov chain on a countable space that is 2-UI (and admits a spectral gap) while it is not hyperbounded. Moreover, we show by example that even hyperboundedness is a weaker property than $\phi$-mixing, which extends considerably the range of models of interest.

What makes the 2-UI condition working is a new efficient version of the Principle of Conditioning [2] that operates with conditional characteristic functions rather than predictable characteristics and leads to the extended functional convergence due to Aldous.

References

1Speaker